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A Family of Classical and Cosserat Continuum Elasto and Rigid-Plasticity Models for Granular Materials

The formulation of continuum mechanical models for the irreversible deformation and flow of granular materials and soils has proven to be a very difficult problem in the sense that the models are mathematically ill-posed but the connection with physical instabilities is not clear cut. The primary physical proper-ties that classically have been included in models are pressure-dependent yield and dilatancy associated with shear flow which does not fully match the pressure dependence. Examples of such models are the ubiquitous plastic potential model and the so-called double-shearing model. It turns out that well-posedness can be obtained by incorporating an additional kinematical quantity representing the pointwise mean grain spin. The fact that well-posed classical continuum models which accurately reflect the physical properties of granular materials have proven elusive to formulate together with the fact that well-posed models can be obtained when the mean-grain spin is incorporated suggests that classical continua are inadequate for granular materials and that higher order continua, such as the Cosserat continuum, provide the best framework for granular materials.

We present a family of such models in which the Cauchy stress is, in general, nonsymmetric and which comprise balance of linear momentum, mass continuity, yield condition (or conditions) and a constitutive equation involving the symmetric part of the stress which generalises the plastic potential model, terms analogous to those contained in the double-shearing motion and also a frame indiferent term involving the velocity vorticity and the mean grain spin. In addition the model comprises the balance of angular momentum, and two constitutive equations, one relating the vorticity and mean grain spin to the assymetric part of the Cauchy stress, the other relating the mean grain spin gradient to the couple stress. As a special case, flows in which the mean grain spin is constant for all space and time may be regarded as flows for a classical continuum and so the full model for a Cosserat continuum contains, a sub-model appropriate for a classical continuum.

We present some properties of the model and, time permitting, consider some simple flows. Finally, we note that the model generalises the standard model for metal plasticity and, wherever possible, we make connections between the models for the two types of material.