Theory of Plates and Shells at the Nanoscale

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The development of nanotechnologies extends the field of application of the classical or non-classical theories of mechanics towards the new materials. The discussions on the application of the continuum mechanics and the mechanics of structures in the nanoscale are very extensive, see [1] among others. In general, modern nanomaterials have physical properties which are different from the bulk material. The classical elasticity can be extended to the nanoscale by implementation of the theory of elasticity taking into account the surface stresses, cf. [1] among others. In particular, the surface stresses are responsible for the size-effect, that means the material properties of a specimen depend on its size. For example, Young's modulus of a cylindrical specimen increases significantly, when the cylinder diameter becomes very small. Let us note that even for such nanostructures as nanoshells, nanofilms, and nanoplates the continuum approach gives a satisfying coincidence with atomistic simulations, if one takes into account the appropriate constitutive equations.

The theory of elasticity with surface stresses was applied to the modifications of the two-dimensional theories of nanosized plates and shells, see, for example, [2–6] and the references in it. The most popular in nanomechanics are the Kirchhoff–Love, Mindlin–Reissner, and von Kàrmàn theories of plate and shells. Here we use the general nonlinear theory of shell presented in [7, 8] for the modification of the constitutive equations taking into account the surface stresses. We show that both the stress and the couple stress resultant tensors may be represented as a sum of two terms. The first term is the volume stress resultant while the second one determined by the surface stresses and the shell geometry. This means that the stress resultants for the shell with surface stresses can be represented as follows

$$\mathbf{T}^* = \mathbf{T} + \mathbf{T}_S\,, \quad \mathbf{M}^* = \mathbf{M} + \mathbf{M}_S\,, \tag{1}$$

where $\mathbf{T}$ and $\mathbf{M}$ are the classical stress and couple stress resultant tensors given for example in [7, 8], while $\mathbf{T}_S$ and $\mathbf{M}_S$ are the resultant tensors induced by the surface stresses, see [6] for details.

In the linear case this modification reduces to the addition of new terms to the elastic stiffness parameters. Follow [4, 5] we show that the bending stiffness is given by

$$D^* = D + D_S\,, \tag{2}$$

where $D = \dfrac{Eh^3}{12(1-\nu^2)}$ is the classical bending stiffness, E and ν are the Young's modulus and the Poisson's ratio of the bulk material, h is the shell thickness, $D_S = h^2\mu^S + h^2\lambda^S/2$, and μ^S and λ^S are the surface elastic moduli.

The influence of the surface stresses on the bending stiffness of a shell is discussed. We show that the surface elasticity makes a shell more stiffer in comparison with the shell without surface stresses, i.e. $D^* > D$. The numerical examples show that the influence of the surface stresses is negligible for the plate thickness more then 20 nm. The effect of the surface stresses may be more significant for multilayered plates and shells and for plates and shells with fractal-like surface.

We also applied the concept of the surface stresses to the plates and shells with rough surface which has fractal-like relief. In particular, we consider the surface coated by nano- or microfibers array and discuss constitutive equations for the effective surface energy of such surfaces. In this case the influence of the surface stresses more significant as in the case of the smooth surface.

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