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Micropolar Media – Theory, Experiments, and Applications

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Most recently Generalized Continuum Theories (GCTs) have gained the attention of the materials science community, the reason being the making of high performance materials with an inner structure for large and small scale applications ranging from light-weight aerospace and automotive panels down to micromechanics and microelectronic gadgets. One of the GCTs is the so-called micropolar theory, which emphasizes the aspect of inner rotational degrees of freedom of a material (see [1] for a modern formulation). Hence this theory seems particularly promising for applications to soils, polycrystalline and composite matter, granular and powder-like materials, porous media and foams and, in particular, to materials that are "somewhere in-between a solid or a fluid," namely liquid crystals.

If we wish to describe the behavior of such materials based on micropolar fluid theory we are confronted with essentially two important schools, the one by Eringen [2,3] and the one by Ericksen-Leslie [4,5]. As we shall initially explain in our presentation both have their merits and both are actually acknowledged by the liquid crystal community, the Ericksen-Leslie theory probably being the most widely used in discussing the nematic state.

In a nutshell we may say that Eringen's approach relies on a consistent use of the original spin balance, whereas Ericksen-Leslie base their technique on a more or less postulated balance for the so-called "intrinsic director force." Moreover, Eringen introduces the concept of the conservation of general micro-inertia whereas Ericksen-Leslie specialize to the description of rigid rod-like molecules, the motion of which is captured by introducing a single vector quantity, the so-called director, $\boldsymbol{d}$. It is therefore not too surprising that both theories lead to different predictions regarding the flow behavior of nematic crystals, although, as we shall demonstrate in analytical and numerical simulations, there are similarities but also adamant dissent. Moreover, due to newly introduced material parameters, the measurement of which is directly linked to the application of either theory, it is not easy to decide, which of the two theories comes closer to a description of reality.

In the case of generalized elastic solids the situation is slightly more relaxed. In this context we face the so-called higher gradient theories of linear elasticity, see [6]. In the case of small deformations this means that we describe the stress-strain relationship not only by the classical Hookean law of linear elasticity, which contains first derivatives of displacement in the linearized strain tensor only. Rather second derivatives appear and also angles of rotation, expressing the micropolar character of the matter, play a role. Consequently, the amount of stiffness parameters required to describe deformation goes beyond Young's modulus and Poisson's ratio and we are confronted with the task to measure additional length-scale dependent parameters. One possibility is to perform bending experiments and to fit the force displacement curves according to the advanced

theory. The higher order stiffnesses can then be obtained by means of an inverse analysis reflecting sometime an increased and sometimes a decreased effective resilience of the materials. In fact, several materials showing internal length scales relevant for their elastic response were investigated by us by using dynamic measurements, Raman spectroscopy, and AFM bending experiments [7,8]: polymers, Aluminum specimens with hole patterns, and foams. We shall report the results and interpret the origin of the length scale.

At this point another remark is in order. Recall that the tensor of the moment of inertia tensor of a continuum particle, $\boldsymbol{J}$, the so-called micro-inertia, plays an important role in context with its rotational degree of freedom, specifically in combination with the angular velocity vector, $\boldsymbol{\omega}$, assigned to the continuum element. In Eringen's as well as in Ericksen-Leslie's fluid theory $\boldsymbol{J}$ is a conserved field quantity, unable of change and production, and not truly an independent variable, such as the mass density, *i.e.*, the inertia of linear momentum, which obeys its own kinetic equation, independently of the momentum balance. Therefore, most recently, it has been emphasized in [6] that the moment of inertia tensor deserves to be treated as a completely independent structural field variable. However, in contrast to the balance of mass, the moment of inertia is not conserved. Rather its governing equation contains a production term, χ_J, which in terms of continuum theory must be considered as a new constitutive quantity. We shall demonstrate in our presentation that it allows to model additional features of materials, *e.g.*, processes with considerable structural changes, such as transitions from an anisotropic to an isotropic state, and *vice versa* (see, *e.g.*, [10]) . In fact, this feature might be useful in order to unite the two theories, or at least point out and remedy their deficiencies, as we shall attempt to do at the end of our talk by giving an outlook into the future.

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